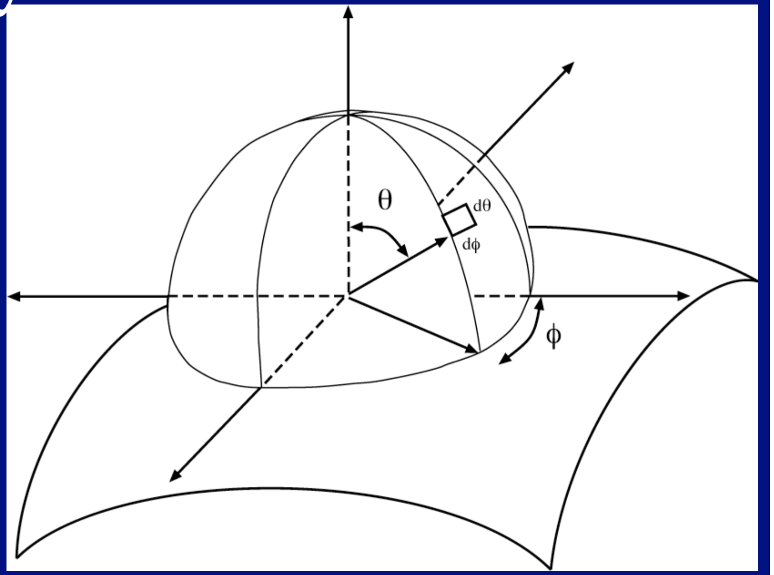
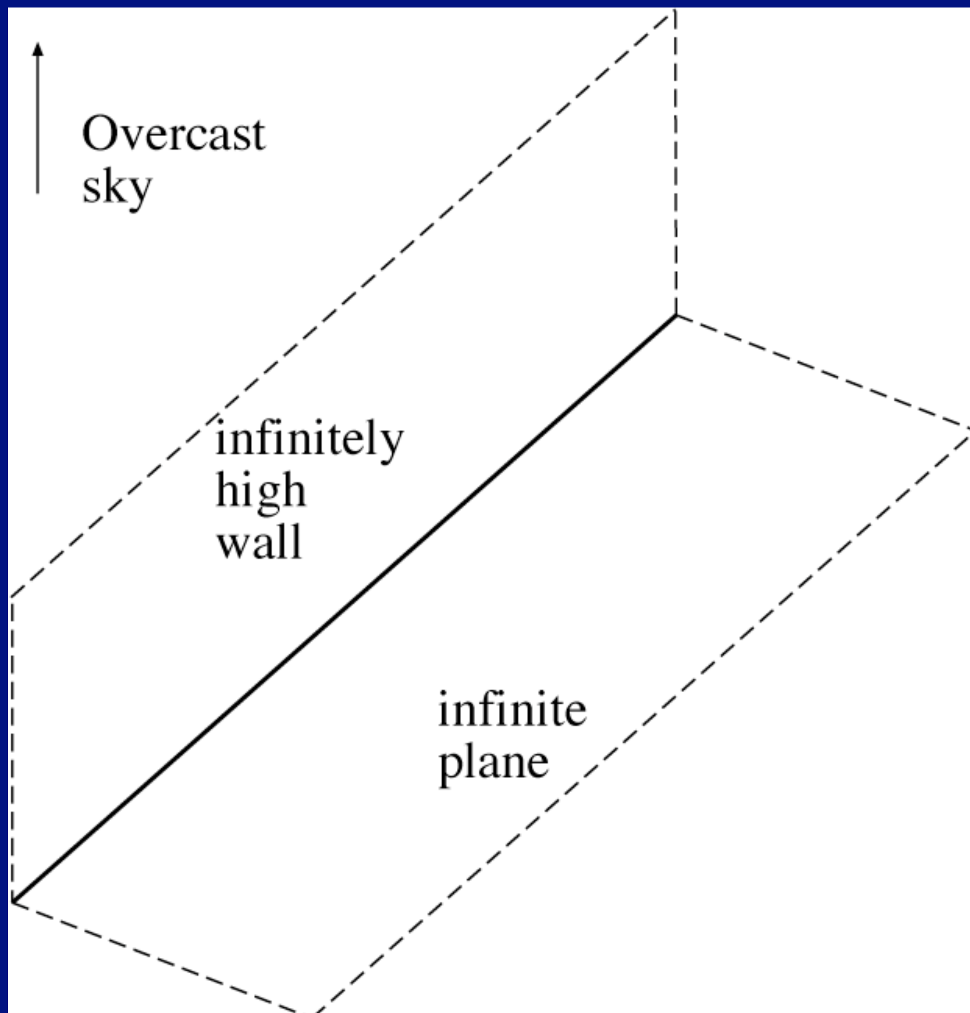
Radiometry

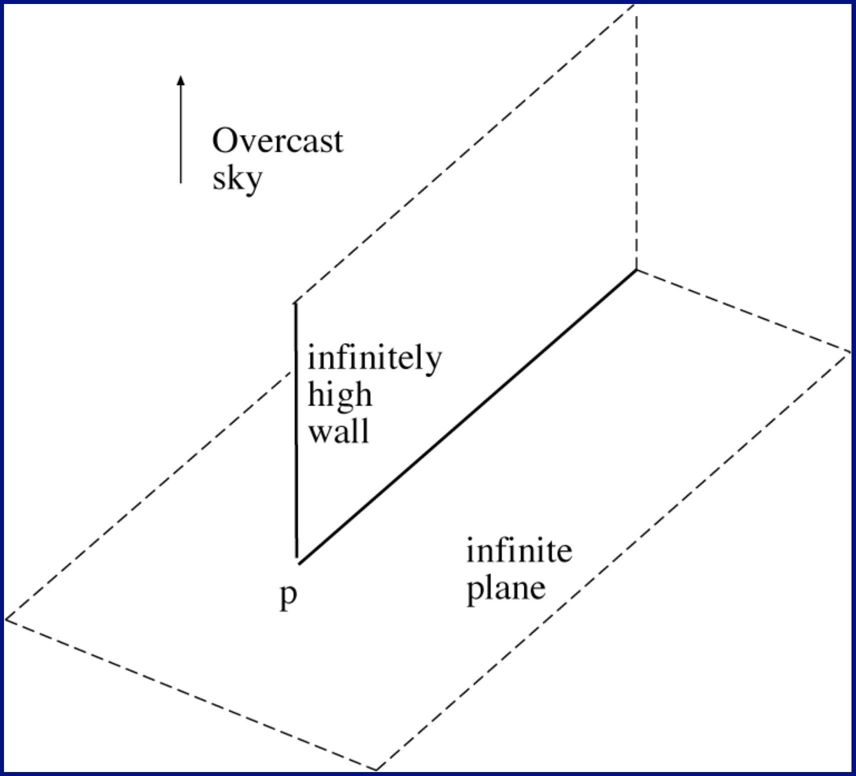


- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

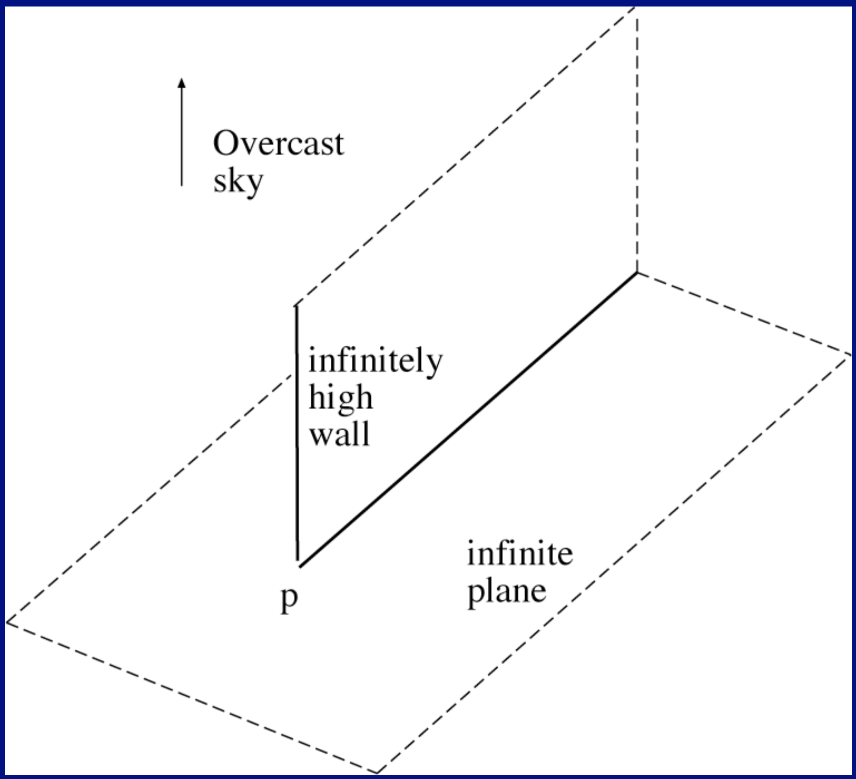
Lambert's wall

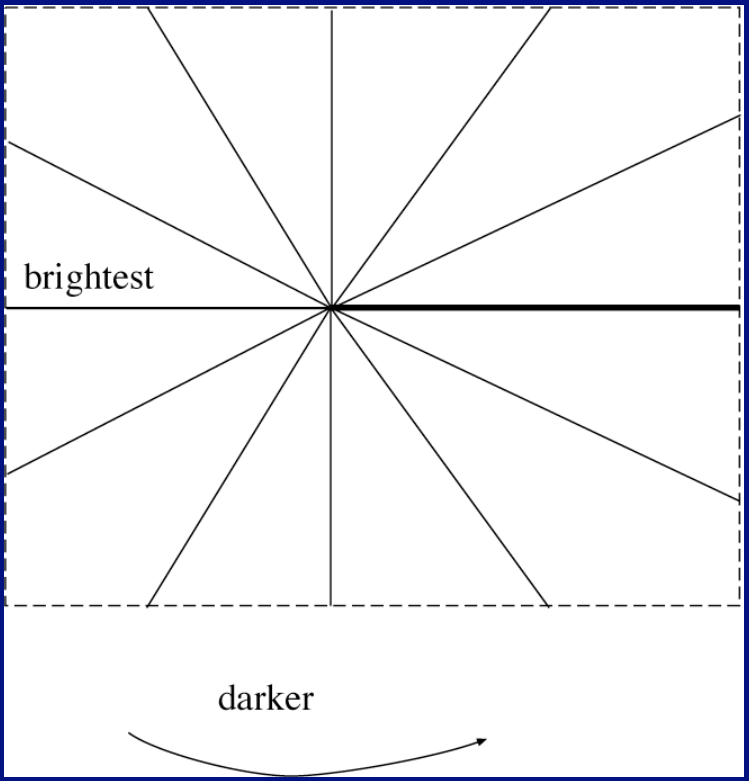


More complex wall



More complex wall

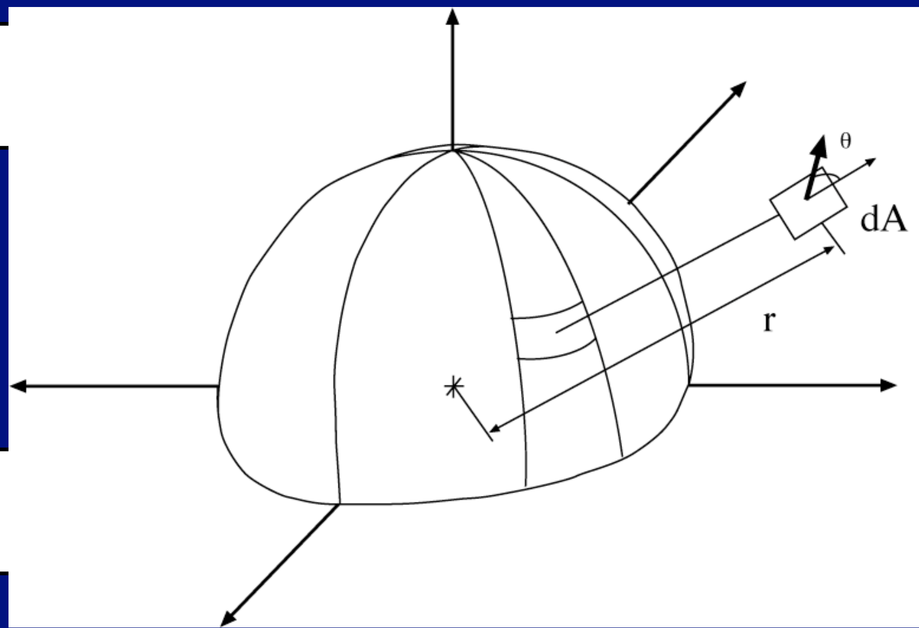




Solid Angle

- By analogy with angle (in radians)
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\vartheta}{r^2}$$



- Another useful expression:

$$d\omega = \sin\vartheta(d\vartheta)(d\phi)$$

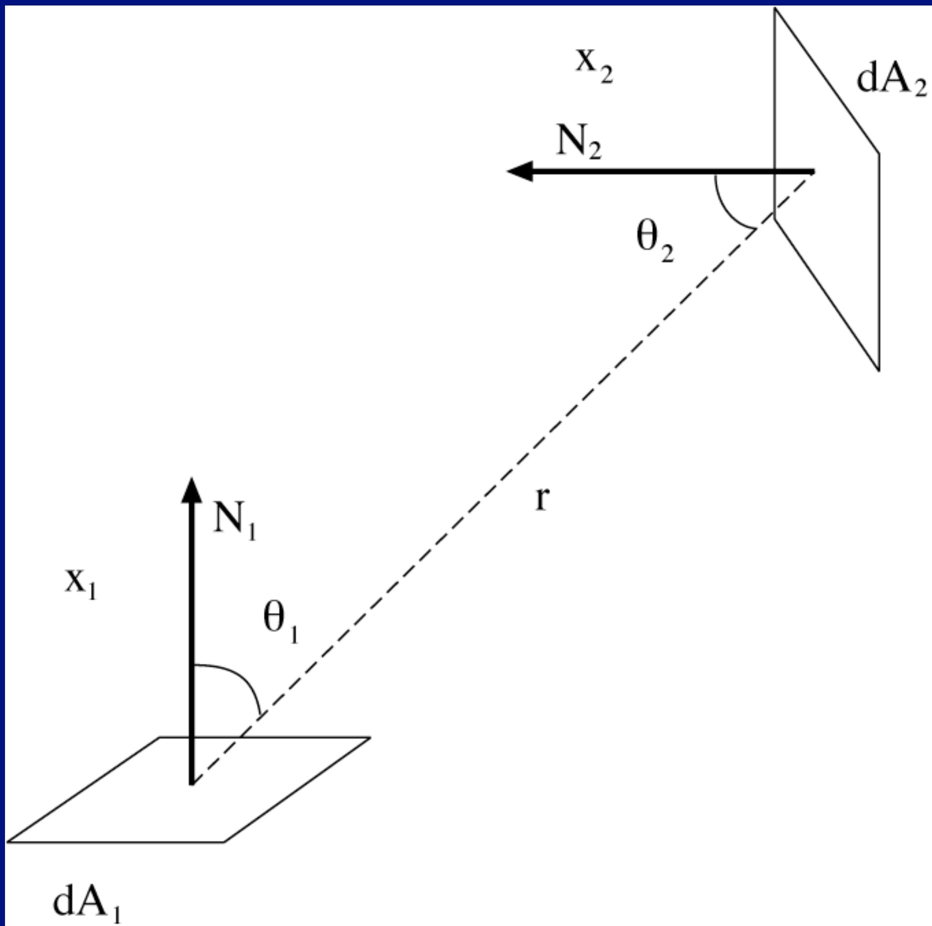
Radiance

- Measure the "amount of light" at a point, in a direction
- Property is:
  Radiant power per unit foreshortened area per unit solid angle
- Units: watts per square meter per steradian (wm-2sr-1)
- Usually written as:

$$L\left(\underline{x}, \vartheta, \varphi\right)$$

• Crucial property:
In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
– hence the units

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

Irradiance

- How much light is arriving at a surface?
- Sensible unit is Irradiance
- Incident power per unit area not foreshortened
- This is a function of incoming angle.
- A surface experiencing radiance $L(x,\theta,\phi)$ coming in from $d\omega$ experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta\sin\vartheta d\vartheta d\varphi$$

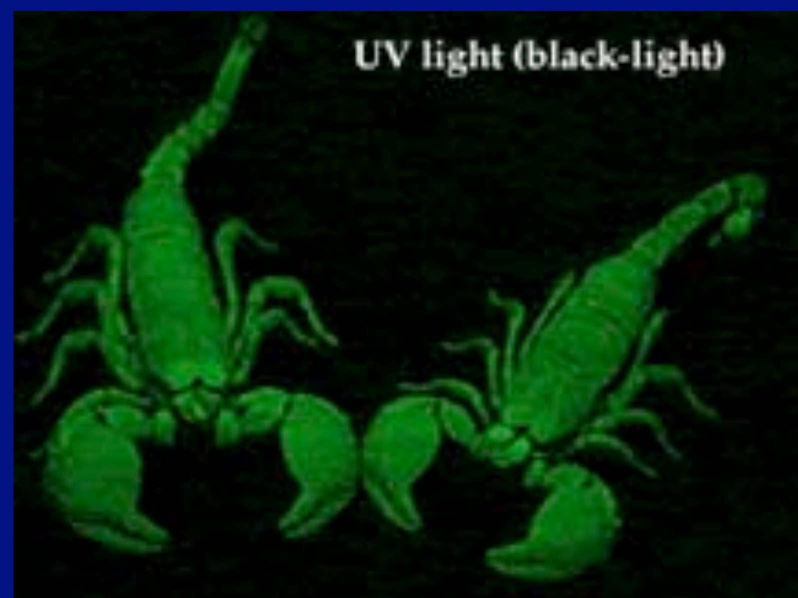
• Crucial property:
Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit

Surfaces and the BRDF

- Many effects when light strikes a surface -- could be:
  - absorbed; transmitted. reflected; scattered
- Assume that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right) = \frac{L_o\left(\underline{x},\vartheta_o,\varphi_o\right)}{L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega}$$


White light (indoor)


UV light (black-light)

BRDF

- Units: inverse steradians (sr-1)
- Symmetric in incoming and outgoing directions
- Radiance leaving in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega} \rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right)L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega_i$$

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm-2)
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions

$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x}, \vartheta, \varphi) \cos \vartheta \, d\omega$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

$$
\begin{aligned}
B(\underline{x}) &= \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega \\
&= L_o(\underline{x})\int_{\Omega} \cos\vartheta d\omega \\
&= L_o(\underline{x})\int_{0}^{\pi/2}\int_{0}^{2\pi} \cos\vartheta \sin\vartheta d\varphi d\vartheta \\
&= \pi L_o(\underline{x})
\end{aligned}
$$

Directional hemispheric reflectance

- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure and very unstable
  - for many surfaces, light leaving the surface is largely independent of exit angle (surface roughness is one source of this property)
- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range 0-1

$$\rho_{dh}(\vartheta_i,\varphi_i) = \frac{\int_\Omega L_o(\underline{x},\vartheta_o,\varphi_o)\cos\vartheta_o d\omega_o}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega_i}$$

$$= \int_\Omega \rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,)\cos\vartheta_o d\omega_o$$

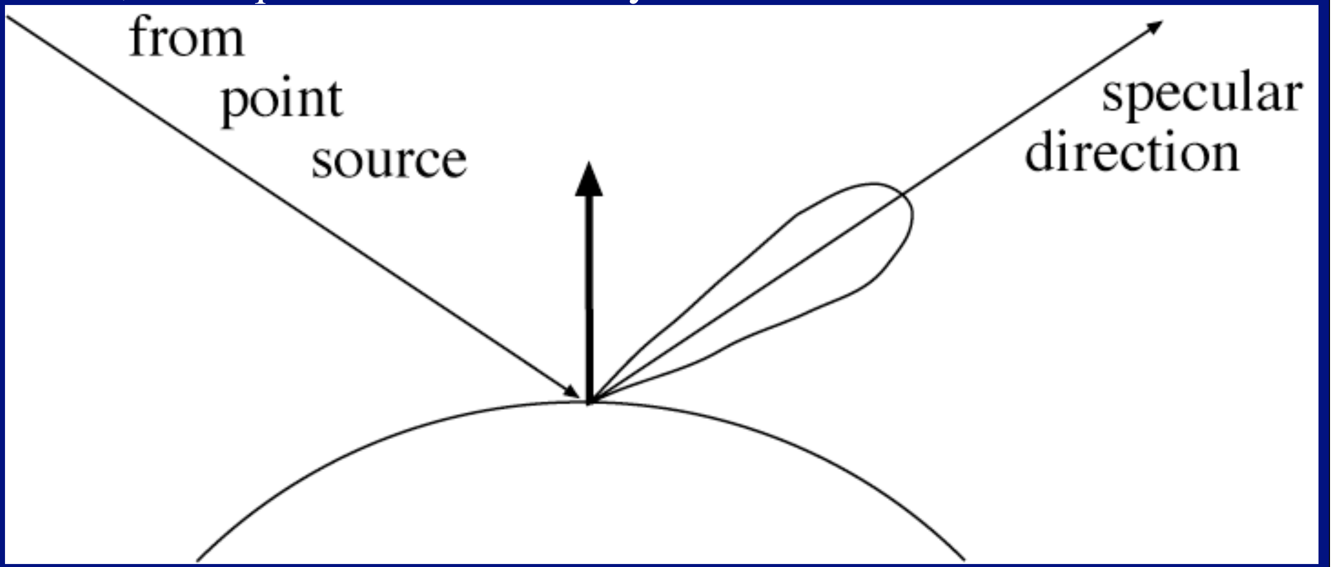
Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of direction
  - cotton cloth, carpets, matte paper, matte paints, etc.
  - radiance leaving the surface is independent of angle
  - Lambertian surfaces (same Lambert) or ideal diffuse surfaces
  - Use radiosity as a unit to describe light leaving the surface
  - DHR is often called diffuse reflectance, or albedo
- for a Lambertian surface, BRDF is independent of angle, too.
- Useful fact:

$$\rho_{brdf} = \frac{\rho_d}{\pi}$$

Specular surfaces

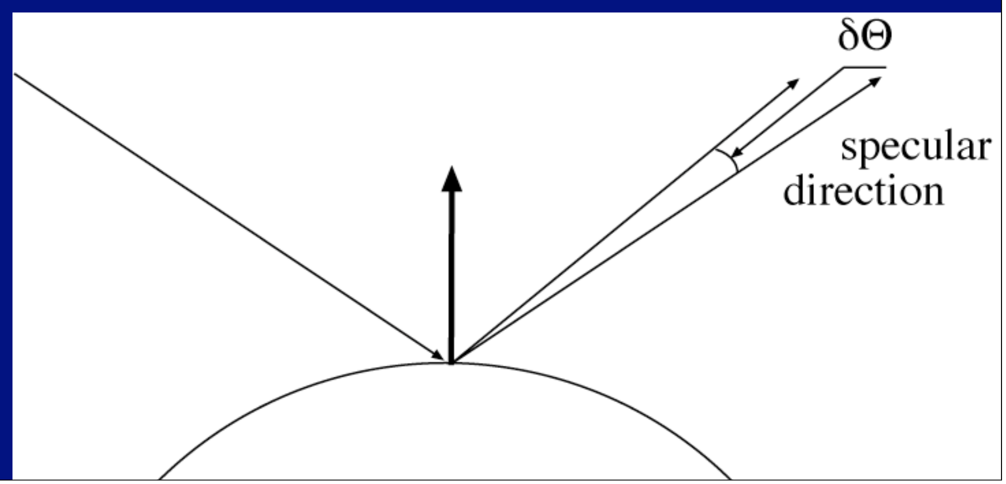
- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Phong's model

- There are very few cases where the exact shape of the specular lobe matters.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as "specularities"
  - very big -- faint specularities
- Phong's model
  - reflected energy falls off with

$$\cos^n(\delta\vartheta)$$



Lambertian + specular

- Widespread model
  - all surfaces are Lambertian plus specular component
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD's, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces